

# POLYTEMPO COMPOSER: A TOOL FOR THE COMPUTATION OF SYNCHRONISABLE TEMPO PROGRESSIONS

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## ABSTRACT

The accurate synchronisation of tempo progressions is a compositional challenge. This paper describes the development of a method based on Bézier curves that facilitates the construction of musical tempo polyphonies up to an arbitrary level of complexity, and its implementation in a software tool. The motivation for this work is to enable and encourage composers to create music with different simultaneously varying tempos which otherwise would be too complex to manage.

## 1. INTRODUCTION

Computers have become an important tool for composers, but very few commercial music applications allow for the generation of multiple simultaneous time streams. This is understandable, as most music of our culture takes place in one single time stream and polytemporal music is quite uncommon. Hence, there is a very small demand for such applications. Composers who nevertheless want to engage in tempo polyphony have to resort to computer music programming environments and implement their own tools, as Dobrian shows exemplarily for the programming environment Max [1]. Yet not every composer is at the same time a skilled programmer and without the availability of appropriate tools, only few composers can create polytemporal music and explore the field of tempo polyphony.

Whereas it is a easy task to construct a succession of musical events that change in tempo just by gradually increasing or decreasing the events' inter-onset intervals, the calculations for tempo progressions that eventually arrive at a defined point in time are not trivial. Nevertheless, these calculations are an essential prerequisite for the synchrony between simultaneous voices in polytemporal music; they are needed, for instance, to construct convergence points of time streams at which musical events of different polyphonic layers converge after a section of independent changes in tempo. Different ways to approach this problem are object of previous studies in the field [2, 3].

This paper proposes a tool to devise temporal polyphonies aimed at composers without in-depth mathematical knowledge. It provides a GUI for the intuitive manip-

ulation of complex tempo structures and the facility to audition the tempo progressions. Another important feature of this tool is its metrical flexibility: It does not restrict the user to regular bar and beat patterns, which is a shortcoming of many similar tools. Tempo polyphonies created with this tool can be exported in different formats to synchronise performing musician or for further use in other computer assisted composition environments.

## 2. MUSICAL CONTEXT

The early 20th century witnessed the rise of many novel compositional techniques to handle musical time. By employing complex, asymmetrical or 'free' rhythms, most of these techniques were intended to overcome musical metre that had dominated western culture for many centuries. Some composers even attempted to negate the traditional notion of a common unifying tempo at all and divided the music into several layers to be played by different musicians in different tempos. Early examples of such music can be found among the compositions of Charles Ives.

### 2.1 Music in Need of Technology

Between 1916 and 1919 Henry Cowell wrote his book *New Musical Resources*, in which he unfolded his theories about the relation of rhythms, metres and tempos to the ratios of the overtone series [4]. He admitted that most of the rhythms he described were too complex to be played by a musician, but he suggested that they be cut on a player piano roll. Cowell never used such a self-playing piano himself, but his book inspired Conlon Nancarrow to compose his *Studies for Player Piano*, which up to the present day still constitute the most substantial corpus of polytemporal music. However, this music is entirely mechanical and overcomes the limitations of human performers by avoiding them altogether.

Polytemporal composition that are to be played by human musicians (i. e. conceived for acoustical instrument that have to be played by human musicians) require a performance aid to keep the tempo. Metronomes or click tracks are commonly used in such situations to allow for a timing accuracy beyond the capabilities of a human performer. In a similar vein one could argue that composers also might benefit from a mathematical aid to devise intricate tempo relationships and tempo progressions whenever the degree of complexity is beyond their mathematical capacity. In

this sense, the focus is put on the composer’s need of technological aids in the remainder of this paper.

## 2.2 Compositional Approaches

There exist different compositional approaches to handle polyphonic tempo structures. Many works deal with the superposition of different independent musical layers. No matter whether these layers are to be played freely or marked with a precise metronome indication, the exact coincidence of any musical events cannot be predicted, because, due to the inaccuracy of the human performance, the tempos will always drift apart to a certain extent. It might be argued that this kind of music is better described as ‘multi-layered’ than truly polyphonic. Needless to say that composers have always been aware of this unavoidable rhythmic blurring and have organised the harmony, melody, gesture etc. of their music accordingly.

Another compositional approach consists in creating a polytemporal music that does not only rely on the expressive quality of stratification *per se*, but is based on a tight synchronisation of all musical events as, for instance, the aforementioned *Studies* by Nancarrow. It must be pointed out that such a temporal precision is not only an issue of rhythm, but also a subject of harmony, as harmony itself is based on the synchronicity of pitched musical events.

What is required to compute the accurate timing of all musical events in a polytemporal counterpoint? The superposition of different tempos (especially in simple ratios) is trivial, but the superposition of different tempo *progressions* (i. e. accelerandos or decelerandos) is a difficult task. The difficulties lie not only in the intricacies of the mathematic formulas, but also in the not entirely obvious and intuitive interdependence of the parameters involved, such as elapsed time, note value, tempo and a specific change in tempo (given as a characteristically shaped tempo curve).

## 3. DEFINITIONS

### 3.1 Tempo

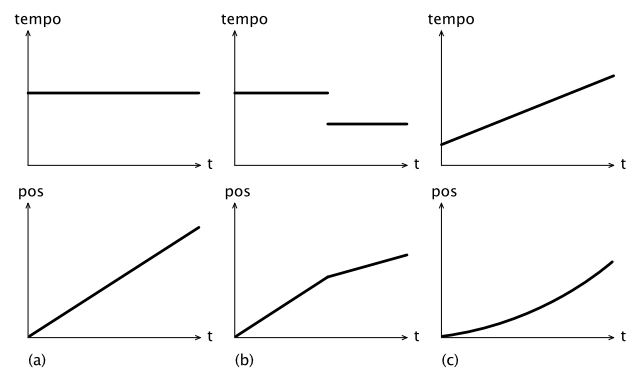
One of the first things a novice musician must learn is a sense for a regular pulse. The ability to accurately perform a (notated) rhythm, no matter how complicated or asymmetrical it might be, is based on this skill. This notion of a regular pulse is deeply ingrained in our musical practice, thus we know the concept of *beat* and assume that it is a constant unit of time. Hence, *tempo* is usually defined as rate of the pulse or beat, measured in beats per minute. This definition, however, is problematic for our purposes for two reasons: First, the unit of the beat is not explicitly defined, which would be necessary in order for the tempo to be handled mathematically. Second, even though the concept of a constant unit of time is a basic principle of music making, this definition rules out metrical structures with non-isochronous pulses, i. e. asymmetrical metres or changes between simple and compound metres. A more open definition of tempo avoids the reference to a countable beat: tempo is *note value per time unit*. For example, one crotchet per second is  $\text{tempo} = 0.25s^{-1}$ .

### 3.2 Score Position

The position in the score is often referred to as ‘symbolic time’ or ‘score time’, which both are unfortunate terms, as the note values of the symbolic music notation only express duration *ratios*, which only become actual physical durations only when executed in a certain tempo. Therefore, the terms *score position* or *score distance* are preferred. The timing of every sonic event is defined by a location in time and a position in the score. Just as time is continuously incremented, the source position can be measured from an established zero-point as well, e. g. the second crotchet in the tenth  $\frac{4}{4}$  bar is at a score position of  $9\frac{1}{4}$  etc.

### 3.3 Tempo Map and Time Map

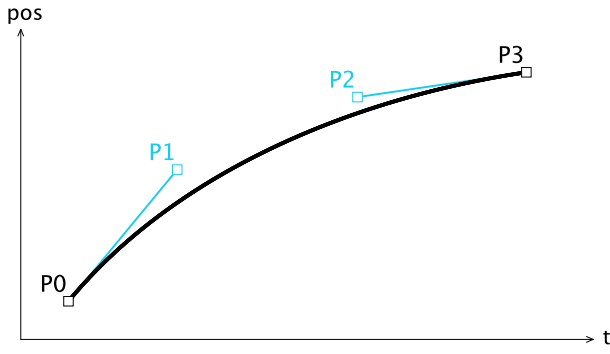
There are two graphical representations of musical time: the *tempo map* and the *time map*. The tempo map depicts speed as function of time, the time map score position as function of time (see Fig. 1). In either representation one parameter always remains implicit: In the tempo map the score position is the integral of the curve, in the time map the tempo is the derivative of the curve.



**Figure 1.** Three tempo maps and their equivalent representation as time maps; (a) depicts a constant tempo, (b) a sudden tempo change, (c) an accelerando.

The tempo map seems to be the most natural way to describe a tempo progression. It is also the starting point for most methods to calculate varying tempos. The time map was introduced by Jaffe [5] as means to realise expressive timing while maintaining overall synchronisation. It is mainly used in research into performance practice to describe timing deviations or rubato, as function to associate ‘idealised’ time to ‘actual’ time [6–8].

The objective of the method presented in this paper is not the formalisation of expressive devices such as rubato, but the synchronisation of individual tempo progressions. As soon as it comes to synchronisation, all calculations based on the tempo curve face an intricate problem: The alignment of two musical events (e. g. two downbeats) in two different independent tempo streams does nearly always require a certain amount ‘error correction’, i. e. a function that distorts the tempo curve appropriately. When using a time map, on the other hand, the synchronisation of events is obvious and as easy as the horizontal alignment of points.



**Figure 2.** A cubic Bézier curve defined by four points. If drawn on a time map,  $P_0$  and  $P_3$  represent assignments of score locations to exact times;  $P_1$  and  $P_2$  control the shape of the curve, especially the slope of the curve at  $P_0$  and  $P_3$  which describes the initial and final tempo.

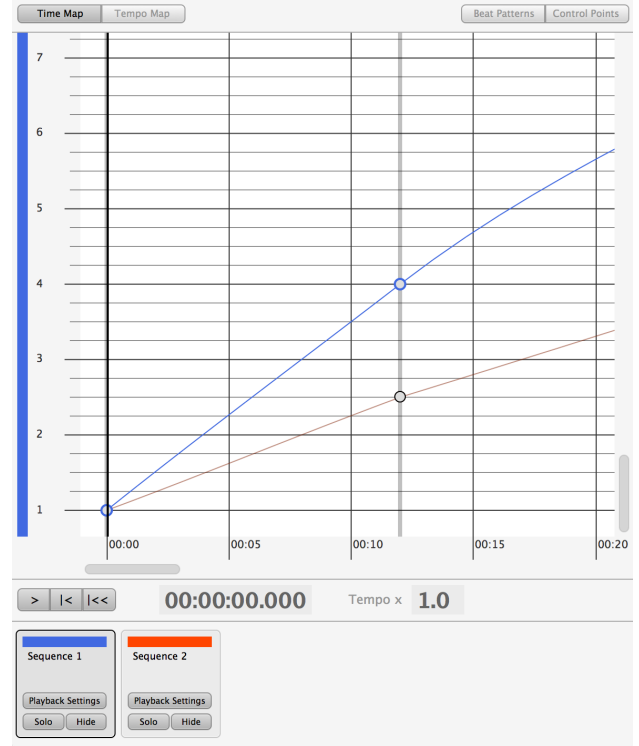
#### 4. MATHEMATICS

For the purpose of synchronisation, it is necessary to assign exact times to certain score positions. These assignments are called *control points* and are set in the time map to build the temporal scaffold of the music on which the timing of all musical events depends. Once these control points are set, they are connected by Bézier curves (a solution already proposed by Berndt [8]). As shown in Fig. 2 a cubic Bézier curve is defined by four points  $P_0$ ,  $P_1$ ,  $P_2$  and  $P_3$ . The curve starts at  $P_0$  and ends at  $P_3$ . The additional points  $P_1$  or  $P_2$  provide directional information and determine the shape of the curve. Their relative position to  $P_0$  and  $P_3$  determine the slope at the beginning and the end of the curve, which in our case represents the initial and final tempo of the section. The distance between  $P_0$  and  $P_1$  or between  $P_2$  and  $P_3$  is used to weight the respective tempo, i. e. to determine how long the initial tempo is held or how soon the final tempo is reached. The time curve (and hence implicitly also the tempo curve) can be warped by moving these two additional points. This can be used, within reasonable limits, to adjust the shape of the tempo progression in order to achieve the desired gestural quality.

#### 5. IMPLEMENTATION

In order to make it readily accessible for composers, this formalism has been implemented in a standalone application. The application is programmed in C++ using the framework JUCE<sup>1</sup>. The user interface is structured as follows: On the topmost level, the temporal polyphony is organised in sequences. Every sequence represents one concurrent tempo stream, i. e. a polyphonic layer or the part of an instrument in a ensemble, when described in terms of a traditional score. Each sequence consists of an event pattern and a list of control points. The event pattern describes the sequence's rhythmical structure and the control points match time and score position (see Fig. 3).

<sup>1</sup> <http://www.juce.com> (URL valid in July 2016)



**Figure 3.** A screenshot of *Polytempo Composer*: two tempo streams (sequences) depicted on a time map. Both streams are in different tempos (expressed by the steepness of the curve) and synchronised at one point around 00:12 seconds.

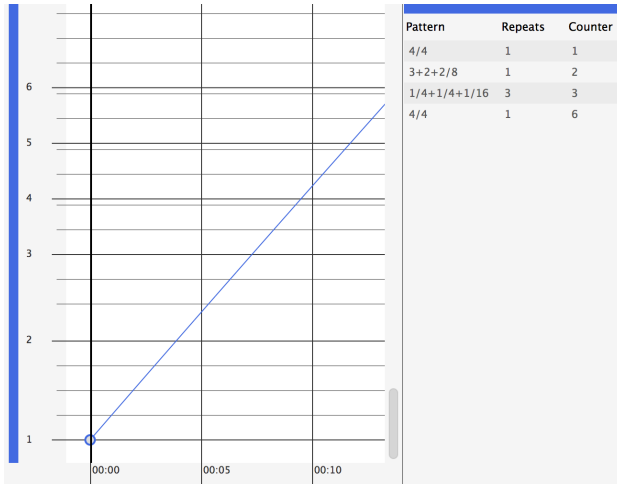
$\frac{4}{4}$	$\frac{3+2+2}{8}$	$\frac{2+1}{4+16}$
4/4	$3+2+2/8$	$1/4+1/4+1/16$
	or	or
	$3/8+2/8+2/8$	$4+4+1/16$

**Figure 4.** The input format of the event pattern is oriented towards the traditional music notation and allows for complex and additive metres.

#### 5.1 The Event Pattern

The event pattern is expressed in score position units. In case of a traditionally conceptualised and notated music, these units can, for the sake of simplicity, be equaled with rhythmical values. If one intends to generate a metronome track, the event pattern is set to correspond to the metrical structure of the music, i. e. every event represents one beat. For convenience, the metres can be entered in the usual way as fraction, whereby a '+' is used to define non-isochronous metres (see Fig. 4). If the music is based on varying metres, the event pattern is composed of several sub-patterns, each of which represents one bar and can be repeated multiple times (see Fig. 5).

If the intended output is not a metronome track, the event pattern does not have to represent a metre. Rather, it can



**Figure 5.** To generate a metronome track, the event pattern corresponds to the bar/beat structure of the music. A list of sub-patterns (on the right) constitute the overall event pattern. The metrical structure is reflected in the marks on the vertical axis of the time map (on the left).

Time	Position	Tempo In	Tempo Out
0	0	---	0.25
10	3	0.25	0.25
20	5	0.25	0.25
40	10	0.25	---

**Figure 6.** List of control points specified by time, position, incoming and outgoing tempo. The tempos are indicated in units per second.

be put together as a list of proportional units to form any arbitrary, periodic or irregular, rhythmic structure.

## 5.2 Control Points

A control point is a precisely defined assignment of a certain score position to a certain time. It can be input graphically directly on the time map or numerically in a table (see Fig. 6). Apart from time and score position each control point is further defined by an incoming and an outgoing tempo whose values determine the slope of the curve at this point. Usually these two tempos are the same. However, in order to express a discontinuity in tempo, e.g. a sudden change of speed or a tempo modulation between two proportional tempos, two different values have to be input. Tempos are specified in units per second. Users who find this counterintuitive can change the tempo format to the familiar ‘crotchets per minute’ in the global settings.

Helper functions are provided to move a control point automatically to a location where it meets certain criteria: A point can be placed, for instance, in such a way that a constant tempo is kept from the previous point on.

## 5.3 Output

The exact time of every event is calculated from the tempo curve that is generated as cubic Bézier curve interpolation

between the control points. The events timed in such a way can be output in various ways.

The events can be played back from within the software using differently pitched notes (like a metronome) and, if desired, each sequence on a different hardware audio channel. This audio playback feature is primarily designed for the composer to listen to any tempo polyphony under construction, which is essential, as composers need their ears, rather than their eyes, to judge if a tempo progression does work. Likewise, this feature can be used to generate a click track in real time. Furthermore, every event can be assigned a MIDI or a OSC message, which opens up possibilities for various live performance settings.

For further use in any computer assisted composition environment, a list of all calculated times can be exported as plain text (optionally comma-separated or enclosed in brackets). In case the events represent the metrical structure of the music, the data can be written to a JSON file specifically formatted to be seamlessly transferred to the associated virtual conducting software *PolytempoNetwork* [9].

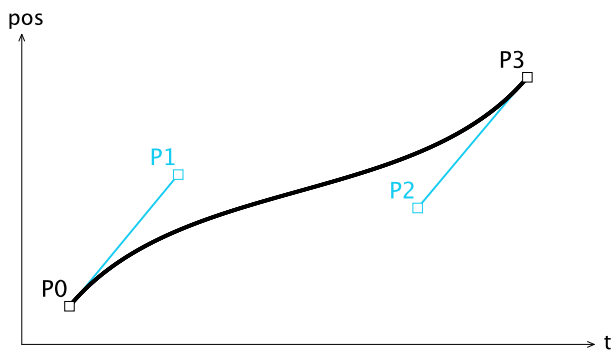
## 6. DISCUSSION

Synchronising tempo progressions is a difficult task. But it is a prerequisite to control temporal polyphonies that consists of more than a stratification of only loosely coordinated layers of music. It is in fact unimaginable that composers conceive a tightly controlled tempo polyphony without the aid of a mathematical tool. Moreover, such a tool can, presumably, help the composer to learn about the fundamental mathematical principles of tempo progressions, and to acquire an important experience for the conception of more advanced polytemporal music. Tempo progressions or tempo curves are defined by several characteristics: initial and final tempo, elapsed time, covered score distance and shape of the curve. All these characteristics are interdependent, which the composer can explore through interaction with the tool.

For example, one might decide that a tempo change does not sound right, because an accelerando is, say, ‘too early too fast’. In such a case, changing the shape of the curve is not the only measure to take (and often does even not yield the desired results). Rather, one could also alter the initial or final tempo, drag one of the control points to another vertical or horizontal position (i.e. change its time or score position) or do a combination of all these.

The interdependence of parameters also appears very clearly when one attempts to produce an ‘impossible’ tempo progression. When, for example, the tempo is too fast for a too short a distance in the score, the time curve becomes automatically s-shaped to compensate the timing error (see Fig. 7). The resulting tempo fluctuation depends on the amount of distortion of the curve; it may range from an almost imperceptible rubato to a strong momentary change of speed.

Finally, one point of critique has to be mentioned. As the presented method operates on the time map, it allows for control over the time curve but does provide no (or only implicit) control over the tempo curve. This might be un-



**Figure 7.** The tempo at  $P_0$  and  $P_3$  is the same. However, this tempo is too fast for too short a score distance (or too long a duration). To compensate for this error, the time curve becomes s-shaped.

usual or even unwanted for some composers. However, due to the fact that synchronisation is the first and foremost feature of this method, this seems to be a sensible compromise.

## 7. CONCLUSION & OUTLOOK

This paper described a method to compute synchronisable tempo progressions and its implementation in a software tool that can potentially aid the composer in realising polytemporal music with complex tempo relationships. The presented implementation is sufficiently general to be used in the context of symbolically notated instrumental music as well as electroacoustic music. It facilitates the exploration of complex tempo structures even for composers who do not want to engage with the intricacies of the underlying mathematics.

It is hoped, that composers will make use of this software and thus create different user scenarios that will stimulate the further development and improvement of this software. Apart from being a tool for experimental polytemporal music, this software could also be a useful tool for film music, when ever it is needed to synchronise specific points in the score tightly with the film.

Future developments might also include score typesetting. No existing music engraving software of high quality provides a straightforward facility to notate different tempos at the same time, let alone different varying tempos. A notation tool that allows for the notation of different parts of a score in different tempos would be beneficial for composers of instrumental music.

The software *Polytempo Composer* and its source is freely available for download from the project's website: <http://polytempo.zhdk.ch> (URL valid in July 2016).

## 8. REFERENCES

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